LECTURE 3-5: EXERCISES

(1) You live in a town of $n+1$ people. You study rumors. You start a rumor by telling it to one other person, who then picks a person at random from the town and passes the rumor on. The second person likewise picks a recipient at random and so forth. What is the probability that the rumor you started is told k times and then comes back to you?

(2) In a game you roll a single, six sided, classical die. Every time you roll a 6, 50E is added to your casino account, every time you do not role a 6, 10E is subtracted from your account. At the end of 10 rolls you receive your earnings or pay your debt.
   (a) After 10 throws, how much money do you expect to win/loose?
   (b) During these 10 throws, how much is your account likely to fluctuate?
   (c) If you start with 30E, what is the probability that you will will be broke or in debt after 10 rolls?
   (d) The casino decides that it will no longer allow you to go temporarily in debt: as soon as your account is 0E or less you are broke. What would be the answer to (c) then?

(3) Lets imagine that the Mensa is giving the photo of random Hertha player each time you visit it and take a dessert (we assume that there is 11 different cards).
   (a) How many times should you take a dessert on average to have at least one photo of 5 random players?
   (b) How many times should you take a dessert on average to have at least one photo of each team member?

(4) In a hypothetical population of fish, the cumulative probability of dying is given by:
$$P(x) = \left(\frac{x}{10}\right)^2$$
All fishes are dead at the age of 10.
   (a) what is a probability density function for this population?
   (b) how likely it is that the fish will die between the age 3 and 4?

(5) A particle undergoes a random walk along x-axis. A step to the right has probability p, a step to the left q. Calculate and graph the probability distributions after ten steps if:
   (a) $p = 0.5$
   (b) $p = 0.9$

(6) Consider the example of Michaelis-Menten enzyme kinetics:
$$\begin{aligned} S + E &\rightarrow SE \\ SE &\rightarrow S + E \\ SE &\rightarrow P + E \end{aligned}$$
   (a) represent this example using Petrinet diagram,
   (b) represent it as $N = (P, T, Pre, Post, M)$ assuming that we have $S = 100, E = 20, SE = 0, P = 0$,

(c) calculate matrices S, A,
(d) if the first reaction occurs 20 times, second 10 and last 5 what would be the new state of the system?
(e) can you find another way to reach this state?