Statistical physics

lecture 11

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Theorem

Let P be the transition matrix of a Markov chain, and let u be the probability vector which represents the starting distribution. Then the probability that the chain is in state x_i after n steps is the ith entry in the vector

$$u^{(n)} = uP^n$$

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Fact

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, and it is true

$$\forall k \in \mathbb{N}, l \in \mathbb{N}, k + l = n . u^{(n)} = uP^n = uP^k P^l$$

These equations define a Chapman-Kolmonogov equations for Markov chain.

Chapman-Kolmonogov

Fact

Let $P_i(t)$ be the probability of finding a system S in the state i in the moment t; let $W_{ij}(t, t_0)$ be the probability of transition of a system S from the state j to the state i assuming that the system was in the state j in the moment t_0 and is in the state i in the moment t. Then the Chapman-Kolmonogov equation set is:

$$W_{ij}(t, t_0) = \sum_k W_{kj}(t_1, t_0) W_{ik}(t, t_1)$$

Master equation

Master equation is a set of first-order differential equations describing the time evolution of the probability of a system to occupy each one of a discrete set of states:

$$\frac{dP_i(t)}{dt} = \sum_{k,k \neq i} w_{ik}(t)P_k(t) - w_{ki}(t)P_i(t)$$

where $w_{ik}(t) = \lim_{t-t_1 \to 0} \frac{W_{ik}(t-t_1)}{t-t_1}$

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Introduction to R