Statistical physics
lecture 6

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X

Definition

(The uniform distribution)

$$X \sim Uni(a, b)$$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & otherwise \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b < x \end{cases}$$

X

Definition

(The uniform distribution)

$$E(X) = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

X

Definition

(The normal distribution)

$$X \sim N\left(\mu, \sigma^2\right)$$

$$f_X\left(x\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$F_X\left(x\right) \sim ugly$$

X

Definition

(The normal distribution)

$$E(X) = \mu$$

$$Var(X) = \sigma$$

X

Definition

(Central limit theorem)

If $X_{1,}X_2, ..., X_n$ are independent realisation of an aritrary random quantity with mean μ and variance σ^2 let $\bar{X}_n$ be the sample mean and define $Z_n = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}}$. Then the limiting distribution of $Z_n$is the standard normal distribution: $P(Z_n \leq z) \to N(0,1)$ when $n \to \infty$.

X

Definition

(Approximation of distributions)

$$Bino\left(n,p\right) \approx N\left(npq\right)$$

rule of the thumb: $0.1 \leq p \leq 0.9$ and $n > max\left[\frac{9(1-p)}{p}, \frac{9p}{1-p}\right]$

$$Po\left(\lambda\right) \approx N\left(\lambda,\lambda\right)$$

rule of the thumb: $\lambda > 20$

X

Definition

(Expotential distribution)

$$X \sim Exp\left(\lambda\right)$$

$$f\left(x\right) = \begin{cases} \lambda e^{-\lambda x} & 0 \leq x \\ 0 & otherwise \end{cases}$$

$$F\left(x\right) = \begin{cases} 0 & x < 0 \\ 1 - \lambda e^{-\lambda x} & 0 \leq x \end{cases}$$

X

Definition

(Expotential distribution)

$$E(X) = \frac{1}{\lambda}$$

$$Var(X) = \frac{1}{\lambda^2}$$