Statistical physics

lecture 5

Szymon Stoma

16-11-2009

X

Definition

(The Poisson distribution)

$$X \sim Po\left(\lambda\right)$$

Let us assume:

$$\lambda = E\left(X\right) = np$$

$$S_X = \{1, 2, ...\}$$

$$P\left(X = k\right) = \frac{\lambda^k}{k!} e^{-\lambda}$$

X

Definition

(The Poisson distribution)

$$E(X) = \lambda$$

$$Var(X) = \lambda$$

X

Fact

(Sum of Poisson)

Let $X \sim Po\left(\lambda\right)$ and $Y \sim Po\left(\lambda'\right)$ and X, Y are independent. Then $Z = X + Y$ is $Z \sim Po\left(\lambda + \lambda'\right)$

X

Fact

(Poisson process)

A sequence of time observations is said to follow a Poisson process with rate λ if the number of observations, X, in any interval length t is $X \sim Po\,(t\lambda)$.

X

Definition

(The Geometric distribution)

$$X \sim Geom(p)$$

$$S_X = \{1, 2, ...\}$$

$$P(X = k) = (1 - p)^k p$$
$$P(X \leq k) = 1 - (1 - p)^k$$

X

Definition

(The Geometric distribution)

$$E(X) = \frac{1}{p}$$

$$Var(X) = \frac{1-p}{p^2}$$

X

Definition

(Probability generating function)

Probability generating function is defined as follows:

$$G(s) = \sum_{k \in S_X}^{\infty} s^k P(X = k)$$

where s is a continuous function and P is a distribution of X.
Note, that:

$$G(s)|_{s=1} = 1$$

X

Definition

(1st derivative of prob. gen. fun.)

$$\frac{dG}{ds}\Big|_{s=1} = E\left(X\right)$$

X

Definition

(2st derivative of prob. gen. fun.)

$$\frac{d^2 G}{ds^2}\Big|_{s=1} = E\left(X^2\right) - E\left(X\right)$$

$$Var\left(X\right) = \frac{d^2 G}{ds^2} + \frac{dG}{ds} - \left(\frac{dG}{ds}\right)^2$$

X

Definition

(The uniform distribution)

$$X \sim Uni\left(a, b\right)$$

$$f\left(x\right) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & otherwise \end{cases}$$

$$F\left(x\right) = \begin{cases} 0 & x < a \\ \frac{1}{b-a} & a \leq x \leq b \\ 1 & b < x \end{cases}$$

X

Definition

(The Geometric distribution)

$$E(X) = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

X

Definition

(Expotential distribution)

$$X \sim Exp\left(\lambda\right)$$

$$f\left(x\right) = \begin{cases} \lambda e^{-\lambda x} & 0 \leq x \\ 0 & otherwise \end{cases}$$

$$F\left(x\right) = \begin{cases} 0 & x < 0 \\ 1 - \lambda e^{-\lambda x} & 0 \leq x \end{cases}$$

X

Definition

(Expotential distribution)

$$E(X) = \frac{1}{\lambda}$$

$$Var(X) = \frac{1}{\lambda^2}$$