Statistical physics

lecture 4

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Probability density function

Definition

(Probability density function) If X is a continuous random quantity, then there exists a function $f_X : \mathbb{R} \rightarrow \mathbb{R}$ called the probability density function which satisfies the following:

- $\forall x . f_X(x) \geq 0$
- $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$
- $P(a \leq x \leq b) = \int_a^b f_X(x)\, dx$ for any $a \leq b$

Cumulative distribution function

Definition

(Cumulative distribution function) For continuous case we define CDF as:

$$F(x) = \int_{-\infty}^{x} f_X(z)\, dz$$

Expectation

Definition

(Expectation) The expertctation or mean of a continuous random variable X is called:

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx$$

Variance

Definition

(Variance) The variance of a continuous random variable X is called:

$$Var(X) = \int_{-\infty}^{\infty} (x - E(X))^2 f_X(x)\, dx$$

PDF of linear transformation

Definition

(PDF of linear transformation)
Let X be a continous quantity with PDF $f_X(x)$ and CDF $F_X(x)$ and let $Y = aX + b$. The PDF of Y is given by:

$$f_Y(y) = \left|\frac{1}{a}\right| f_X\left(\frac{y-a}{b}\right)$$

Directed graph

Definition

(Directed graph) A directed graph or digraph, G is a tuple (V, E) where $V = \{v_1, .., v_n\}$ is a set of nodes (or vertices) and $E = \{(v_i, v_j) : v_i, v_j \in V\}$ is a set of direct edges (arcs).

Simple/biparitate graph

Definition

(Simple/biparitate graph) A graph is described as simple if there do not exist edges of the form (v_i, v_i) and there are no repeated edges. A biparitate graph is a simple graph where the nodes are partioned into two distinct subsets V_1, V_2 (i.e. $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$) such thet there are no arcs joining nodes from the same subset.

Petrinet

Definition

(Petrinet) A Petri net, N, is a n-tuple $(P, T, Pre, Post, M)$ where $P = \{p_1, ..., p_u\}$ is a finit set of places (species), $T = \{t_1, ..., t_u\}$ is a finit set of transitions. Pre is a $v \times u$ integer matrix containing the weights of the arcs going from places to transitions (the (i, j)th element of this matrix is the index of the arc going from place j to transition i), and $Post$ s a $v \times u$ integer matrix containing the weights of the arcs going from transitions to places (the (i, j)th element of this matrix is the index of the arc going from transition i to place j). Note that $Pre, Post$ are usually sparse matrices. M is a u-dimensional integer vector representing the current marking of the net (i.e. current state fo the system).

Reaction matrix

Definition

(Reaction matrix) The reaction matrx $A = Post - Pre$ is the $v \times u$ integer matrix whose rows represent the effect of individual transitions (reactions) on the marking (state) of the network. Similartly, the stoichiometry matrix $S = A'$ is the $u \times v$ integer matrix whose columns represent the effect of individual transitions (reactions) on the marking (state) of the network.

Transition rule

Definition

(Transition rule) If r represents the transitions that have taken place subsequent to the marking M, the new marking $\tilde{M}$ is related to the old marking via the matrix equation $\tilde{M} = M + Sr$.