Statistical physics

lecture 3

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Variance

Definition

(Variance) The variance of random quantity X written $Var(X) \equiv \sigma_X^2$ is defined by:

$$Var(X) = \sum_{x \in S_X} \left\{ (x - E(X))^2 P(X = x) \right\}$$

Which is often written:

$$Var(X) = \sum_{x \in S_X} x^2 P(X = x) - E^2(X)$$

Standard deviation

Definition

(Standard deviation) The standard deviation of random quantity X written $SD(X) \equiv \sigma_X$ is defined by:

$$SD(X) = \sqrt{Var(X)}$$

Linear transformation of expectation

Fact

If we have a random quantity X and a linear transformation $Y = aX + b$ where $a, b \in \mathbb{R}$ then:

$$E\left(aX + b\right) = aE\left(X\right) + b$$

Expectation of sum

Fact

For two random variables X, Y we have:

$$E(X+Y) = E(X) + E(Y)$$

Expectation of an independent product

Fact

(Expectation of an independent product) If X and Y are independent random variables, then:

$$E\left(XY\right) = E\left(X\right) E\left(Y\right)$$

Variance of linear transformation

Fact

(Variance of linear transformation) If X is a random quantity with finite variance $Var\left(X\right)$ then:

$$Var\left(aX+b\right)=a^{2}Var\left(X\right)$$

Variance of an independent sum

Fact

(Variance of an independent sum) If X and Y are independent random quantities then:

$$Var\left(X+Y\right)=Var\left(X\right)+Var\left(Y\right)$$

The Bernoulli experiment

Fact

(The Bernoulli experiment)
The probability of single success is equall p. $S_X = \{0, 1\}$

$$E(X) = p$$

$$Var(X) = p(1 - p)$$

The binomial distribution

Definition

(The binomial distribution)
It is a distribution of the number of "successes" in a series of n independent trials, where a "success" comes with probability p (thus a "failure" with probability $1-p$).

$$X \sim B\left(n, p\right)$$

$$P\left(X = k\right) = \binom{n}{k} p^k \left(1-p\right)^{n-k}$$

$$CDF \sim ugly$$

The binomial distribution

$$E\left(X\right) = np$$
$$Var\left(X\right) = np\left(1 - p\right)$$

The discrete uniform distribution

Definition

(The discrete uniform distribution)

$$X \sim DU(X)$$

$$S_X = \{1, 2, ..., n\}$$

$$P(X = k) = 1/n$$

The discrete uniform distribution

$$E(X) = \frac{n+1}{2}$$

$$Var(X) = \frac{n^2 - 1}{12}$$