Statistical physics

lecture 2

Szymon Stoma

26-09-2009

Random quantity/variable

Definition

(Random quantity/variable) A random quantity is a real valued function which acts on elements of the sample space. That is, to each outcome, the random variable assigns a real number.

- $X = x$ means that the observed value of the random quantity X is the number x.
- The set of all possible observed values for X is:

$$S_X = \{X(s) : s \in S\}$$

Probability mass function

Definition

(Probability mass function) For any discrete random variable X we define the probability mass function (PMF) to be the function which gives the probability of each $x \in S_X$:

$$P(\{s \in S : X(s) = x\}) = P(X = x) = \sum_{\{s \in S : X(s) = x\}} P(\{s\})$$

Probability mass function

Definition

The set of all pairs$\{(x, P(X = x)) : x \in S_X\}$ is known as the probability distribution of X.

Cumulative distribution function = CDF

Definition

The CDF is defined by:

$$F_X(x) = P(X < x) = \sum_{\{y \in S_X | y \leqslant x\}} P(X = y)$$

Expectation

Definition

The expectation of random quantity X written $E(X) \equiv \mu_X$ is defined by:

$$E(X) = \sum_{x \in S_X} xP(X = x)$$

Variance

Definition

(Variance) The variance of random quantity X written $Var(X) \equiv \sigma_X^2$ is defined by:

$$Var(X) = \sum_{x \in S_X} \left\{ (x - E(X))^2 P(X = x) \right\}$$

Which is often written:

$$Var(X) = \sum_{x \in S_X} x^2 P(X = x) - E^2(X)$$