Statistical physics

lecture 1

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Probability space

Definition

Probability space is a 3-tuple (S, F, P) where:

- S is a sample space (possible outputs)
- F is an event space (usually it is a set of all subsets of S called 2^S, can be also boreal subset of 2^S)
- P is a probability function ($P : F \to [0, 1]$) and satisfies following conditions:
  - $P(S) = 1$
  - if $A \subseteq S$ then $P(A) \geq 0$
  - if $A \cap B = \emptyset$ then $P(A \cap B) = P(A) + P(B)$

Set operations

Definition

In the event space F "normal" set operation can be performed. Intuitively, results of all these operations are kept in the event space.

Commutative laws:

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

Associative laws:

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Distributive laws:

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

Operations on events

Definition

There is a meaning for different operations performed on sets of events:

- Union of two events $E \cup F$ is the event that at least one of E and F occurs.
- Intersection of two events $E \cap F$ is the event that both of E and F occurs.
- The complement of an event E^C is the event that E does not occur.
- Two events E, F are disjoint (or mutually exclusive) if they can not both occur i.e. $E \cap F = \emptyset$
- The event E is true if the output of experiment s belongs to E i.e. $s \in E$

Classical probability

Definition

(Classical probability) Classical probability theory is concerned with carrying out probability calculation based on equally likely outcomes:

$$P(\{s\}) = \frac{1}{n},$$

where $\#S = n$

Multiplication principle

Definition

(Multiplication principle) If there are p experiments and the first has n_1 equally likely outcomes, the second has n_2 equally likely outcomes, and so on until the pth experiment has n_p equally likely outcomes, then there are $n_1 n_2 ... n_p = \prod n_i$ equally likely outcomes for p experiments.